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On the neutral Higgs boson masses in the MSSM for arbitrary stop mixing

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Abstract

We compute the $\mathcal{O}(\alpha_t\alpha_s)$ two–loop corrections to the neutral Higgs boson masses in the Minimal Supersymmetric extension of the Standard Model. An appropriate use of the effective potential allows us to obtain simple analytical formulae, valid for arbitrary values of m_A and of the mass parameters in the stop sector. We elucidate some subtleties of the effective potential calculation, and find full agreement with the numerical output of the existing diagrammatic calculation. We discuss in detail the limit of heavy gluino.

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1 Introduction

There is a crucial prediction of the Minimal Supersymmetric extension of the Standard Model, or MSSM (for reviews and references, see e.g. ref. [1]), subject to decisive tests at present and future colliders. It is the existence of a light CP–even neutral Higgs boson, h, accompanied by other states $(H, A, H^{\pm})$, whose masses are strongly correlated but can vary over a wide range near the weak scale. An important step in the understanding of the MSSM Higgs sector was the realization that the classical bound $m_h < m_Z$, and, more generally, the classical relations among the gauge and Higgs boson masses, are violated by large radiative corrections, dominated by top and stop loops [2, 3, 4]. After that, extensive efforts have been devoted to progressive refinements of the theoretical predictions for the Higgs boson masses and couplings, as functions of the relevant MSSM parameters. These activities have been performed in several directions, with special emphasis on the prediction for m_h: inclusion of stop mixing effects [5, 6]; resummation of large logarithms using appropriate one– and two–loop renormalization group equations (RGE) [5, 7, 8, 9]; complete one–loop diagrammatic calculations including momentum-dependent effects [10, 11]; calculations of the most important two–loop contributions [12, 13, 14, 15]. Other studies have been oriented towards a meaningful combination of the above results [15, 16], and towards the implementation of the latter in computer codes [17, 18], to be used in turn for experimental analyses [19].

In the present paper we address once more the computation of the neutral Higgs boson masses, whose present state–of–the–art can be summarized as follows. There is a diagrammatic two–loop computation, including $\mathcal{O}(\alpha_t \alpha_s)$ effects, performed for arbitrary m_A and arbitrary values of the parameters in the stop mass matrix, in the zero-momentum limit [13]. While, for small stop mixing and universal soft stop masses, sufficiently simple and accurate analytical formulae have been obtained [14], in the general case the complete formulae are rather lenghty, which may be a problem for their practical implementation in computer codes. The results of ref. [13] can be improved by including the logarithmic $\mathcal{O}(\alpha_t^2)$ corrections, as extracted by solving perturbatively the appropriate RGE [8, 9]. There is also a computation [15] of both $\mathcal{O}(\alpha_t \alpha_s)$ and $\mathcal{O}(\alpha_t^2)$ two–loop corrections to m_h, based on the effective potential approach. This computation, however, is applicable only for $m_A \gg m_Z$. Moreover, the full results of [15] for m_h are available only in numerical form, and accurate and simple analytical formulae were provided under the additional assumptions of small stop mixing and universal soft stop masses.

In view of the situation described above, there is still room for a number of useful improvements that could be achieved without excessive effort. As a first step, one should aim at simple analytical formulae for the two–loop corrected mass matrix of the neutral CP–even Higgs sector, still in the zero–momentum limit and including only $\mathcal{O}(\alpha_t \alpha_s)$ corrections, but for arbitrary values of m_A and of the parameters of the stop mass matrix. One could then proceed with the inclusion of the $\mathcal{O}(\alpha_t^2)$ corrections, and of the corrections coming from the momentum–dependent part of the two–loop Higgs propagators, into the above framework. Finally, one could address the resummation of the large logarithms of $(m_{\tilde{t}_1}/m_{\tilde{t}_2})$, the ratio of the two stop mass eigenvalues, by means of suitable RGE, defined in an appropriate effective theory. This has been done [9] in the case of small stop mixing, but is considerably more complicated in the case where a large splitting between $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ is induced by a large mixing term in the stop mass matrix.

In this paper we accomplish the first step of the above program, leaving the remaining

steps for future work. The paper is organized as follows. After this introduction, section 2 recalls the general features of the calculation of the MSSM neutral Higgs boson masses in the effective potential approach. Section 3 describes the main features of our two–loop calculation of the $\mathcal{O}(\alpha_t\alpha_s)$ contributions, and presents its results, in a form that allows to assign the input parameters either in the $\overline{\rm DR}$ scheme or in some on–shell scheme. In the concluding section we compare our results with the existing literature, and discuss in detail the heavy–gluino limit. This limit requires some care, especially when the input parameters are assigned in the $\overline{\rm DR}$ scheme, as often done in models that predict the soft supersymmetry–breaking masses.

Technical details are confined to three appendices. Appendix A gives the analytical expressions for the two–loop contributions to the neutral Higgs mass matrices that are controlled by the gluino mass. Appendix B gives the explicit formulae that are needed for the transition from the $\overline{\rm DR}$ scheme to our implementation of the on–shell scheme. Appendix C gives the relation between m_3^2 and m_A^2, which may be useful for discussing models that predict the values of the soft supersymmetry–breaking masses, and in particular of m_3^2, at some cut–off scale for the MSSM.

2 Higgs masses in the effective potential approach

The MSSM neutral Higgs boson masses can be identified with the zeros of the corresponding two-point functions, which depend on the external momentum and have in general the form of a matrix. In the limit of vanishing external momentum, these masses can be formally obtained by the following method: compute the effective potential $V_{\rm eff}$, retaining its complete dependence on the neutral Higgs fields, H_1^0 and H_2^0; minimize $V_{\rm eff}$ to find the vacuum expectation values $\langle H_1^0\rangle \equiv v_1/\sqrt{2}$ and $\langle H_2^0\rangle \equiv v_2/\sqrt{2}$; expand $V_{\rm eff}$ around its minumum up to quadratic fluctuations, and diagonalize the resulting mass matrix. Of course, at each step we can carry out explicitly only the calculations that contribute to the final results, at the desired level of approximation. Before moving to our specific two–loop computation, we give now some formulae that illustrate the formal results of the general procedure, and are valid at every order in perturbation theory.

Putting all the other fields to zero, and keeping only the dependence on the neutral Higgs fields, the tree–level Higgs potential of the MSSM reads:

$$V_0 = m_1^2 \left|H_1^0\right|^2 + m_2^2 \left|H_2^0\right|^2 + m_3^2 \left(H_1^0 H_2^0 + {\rm h.c.}\right) + \frac{g^2+g'^2}{8}\left(|H_1^0|^2 - |H_2^0|^2\right)^2 , \qquad (1)$$

where: $m_1^2 = m_{H_1}^2 + |\mu|^2$, $m_2^2 = m_{H_2}^2 + |\mu|^2$; μ is the Higgs mass term in the superpotential; $m_{H_1}^2$, $m_{H_2}^2$ and m_3^2 are soft supersymmetry–breaking masses; g and g' are the $SU(2)_L$ and $U(1)_Y$ gauge couplings, respectively. It is not restrictive to choose m_3^2 real and negative, so that v_1 and v_2 are real and positive, and the neutral Higgs fields can be decomposed into their vacuum expectation values plus their CP–even and CP–odd fluctuations as follows:

$$H_1^0 \equiv \frac{v_1 + S_1 + i\,P_1}{\sqrt{2}}\,, \qquad H_2^0 \equiv \frac{v_2 + S_2 + i\,P_2}{\sqrt{2}}\,. \qquad (2)$$

In the effective potential approach, the mass matrices for the neutral CP–odd and CP–even Higgs bosons can be approximated, at every order in perturbation theory, by:

$$\left(\mathcal{M}_P^2\right)_{ij} = \left.\frac{\partial^2 V_{\rm eff}}{\partial P_i \partial P_j}\right|_{\rm min}, \qquad \left(\mathcal{M}_S^2\right)_{ij} = \left.\frac{\partial^2 V_{\rm eff}}{\partial S_i \partial S_j}\right|_{\rm min}, \qquad (i,j=1,2)\,, \qquad (3)$$

where $V_{\rm eff} = V_0 + V$ is the loop–corrected Higgs potential in the $\overline{\rm DR}$ scheme, and $\langle S_1\rangle = \langle P_1\rangle = \langle S_2\rangle = \langle P_2\rangle = 0$. Using the explicit expression of the tree–level potential, eq. (1), v_1 and v_2 are determined by minimizing the effective potential:

$$\frac{1}{v_1}\,\frac{\partial V_{\rm eff}}{\partial S_1}\bigg|_{\rm min} \;=\; m_1^2 + m_3^2\,\frac{v_2}{v_1} + \frac{(g^2+g'^2)}{4}(v_1^2-v_2^2) + \frac{1}{v_1}\frac{\partial V}{\partial S_1}\bigg|_{\rm min} \;=\; 0\,, \tag{4}$$

$$\frac{1}{v_2}\,\frac{\partial V_{\rm eff}}{\partial S_2}\bigg|_{\rm min} \;=\; m_2^2 + m_3^2\,\frac{v_1}{v_2} + \frac{(g^2+g'^2)}{4}(v_2^2-v_1^2) + \frac{1}{v_2}\frac{\partial V}{\partial S_2}\bigg|_{\rm min} \;=\; 0\,. \tag{5}$$

Combining eqs. (1)–(5), the CP–odd and CP–even Higgs mass matrices become ($i,j=1,2$):

$$\left(\mathcal{M}^2_P\right)_{ij} \;=\; -m_3^2\,\frac{v_1 v_2}{v_i v_j} - \frac{\delta_{ij}}{v_i}\frac{\partial V}{\partial S_i}\bigg|_{\rm min} + \frac{\partial^2 V}{\partial P_i \partial P_j}\bigg|_{\rm min}\,, \tag{6}$$

$$\left(\mathcal{M}^2_S\right)_{ij} \;=\; (-1)^{i+j}\left[-m_3^2\,\frac{v_1 v_2}{v_i v_j} + \frac{(g^2+g'^2)}{2}v_i v_j\right] - \frac{\delta_{ij}}{v_i}\,\frac{\partial V}{\partial S_i}\bigg|_{\rm min} + \frac{\partial^2 V}{\partial S_i \partial S_j}\bigg|_{\rm min}\,. \tag{7}$$

Combining further eqs. (6) and (7), we can write the CP–even Higgs mass matrix as follows:

$$\left(\mathcal{M}^2_S\right)_{ij} = (-1)^{i+j}\left[\left(\mathcal{M}^2_P\right)_{ij} - \frac{\partial^2 V}{\partial P_i \partial P_j}\bigg|_{\rm min} + \frac{(g^2+g'^2)}{2}v_i v_j\;\right] + \frac{\partial^2 V}{\partial S_i \partial S_j}\bigg|_{\rm min}\,, \tag{8}$$

where the CP-odd mass matrix can be expressed, in terms of the loop–corrected CP-odd Higgs mass m_A and of $\tan\beta = v_2/v_1$, as:

$$\mathcal{M}^2_P = \left(\begin{array}{cc} \sin^2\beta & \sin\beta\,\cos\beta \\ \sin\beta\,\cos\beta & \cos^2\beta \end{array}\right) m_A^2\,. \tag{9}$$

3 $\mathcal{O}(\alpha_t\alpha_s)$ two-loop corrections to the neutral Higgs boson masses

The formulae derived in the previous section have general validity, and were employed long ago for the one–loop computation [2, 3, 5, 6]. We will now follow the same strategy for the calculation of the $\mathcal{O}(\alpha_t\alpha_s)$ two–loop corrections to the entries of the neutral CP–even Higgs boson mass matrix. The relevant Feynman diagrams involve top, stop, gluons and gluinos on the internal lines, and are shown in figure 1. Since $V_{\rm eff}$ must be considered in a generic Higgs background, it is important to elucidate the dependence of the propagators and vertices on the Higgs fields.

In a generic Higgs background, the MSSM Lagrangian contains the following bilinear terms in the top fields:

$$\mathcal{L}_{2t} = (\overline{t'_L}\;\;\overline{t'_R})\left(\begin{array}{cc} i\,\not\partial & -X^* \\ -X & i\,\not\partial \end{array}\right)\left(\begin{array}{c} t'_L \\ t'_R \end{array}\right), \tag{10}$$

where t'_L and t'_R are four–component fermions of definite chirality, and the field–dependent mixing term X is:

$$X \;=\; h_t\,H_2^0 \;\equiv\; |X|\,e^{i\,\varphi}\,, \qquad\qquad (0\leq\varphi<2\pi)\,. \tag{11}$$

It is not restrictive to assume that h_t is real and positive, then $\langle|X|\rangle = h_t v_2/\sqrt{2}$ and $\langle\varphi\rangle = 0$. Analogously, the terms quadratic in the top squarks are:

$$\mathcal{L}_{2\tilde t} = -(\tilde t'^*_L\;\;\tilde t'^*_R)\left(\begin{array}{cc} \Box + m_L^2 & \widetilde X^* \\ \widetilde X & \Box + m_R^2 \end{array}\right)\left(\begin{array}{c} \tilde t'_L \\ \tilde t'_R \end{array}\right), \tag{12}$$

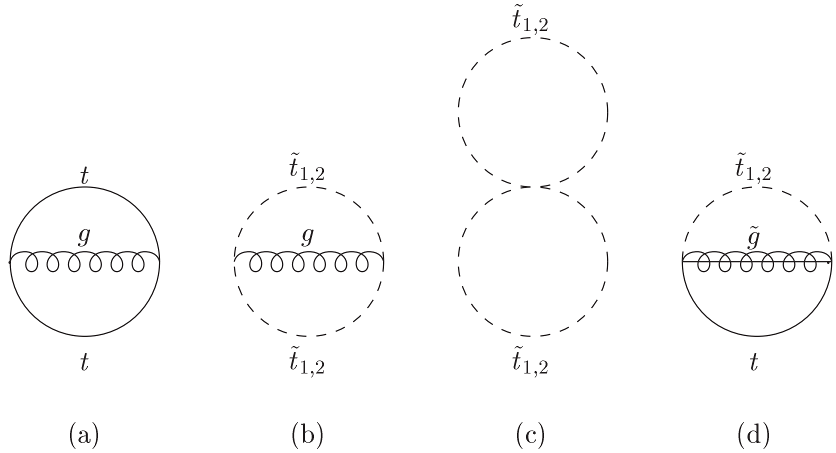

Figure 1: Diagrams that contribute to the two–loop effective potential and affect the $\mathcal{O}(\alpha_t\alpha_s)$ calculation of the neutral Higgs boson masses.

and the field–dependent entries of the stop mass matrix are, neglecting D–term contributions that vanish for $g = g' = 0$:

$$m_L^2 = m_Q^2 + h_t^2\,|H_2^0|^2\,, \qquad m_R^2 = m_U^2 + h_t^2\,|H_2^0|^2\,, \tag{13}$$

$$\widetilde{X} \equiv |\widetilde{X}|\,e^{i\,\widetilde{\varphi}} = h_t\,\left(A_t H_2^0 + \mu H_1^{0\,*}\right)\,, \qquad (0 \leq \widetilde{\varphi} < 2\pi)\,, \tag{14}$$

where m_Q^2, m_U^2 and A_t are the soft supersymmetry–breaking mass parameters of the stop sector. We assume here μ and A_t to be real, so that $\langle|\widetilde{X}|\rangle = (h_t v_2/\sqrt{2})\,|A_t + \mu\cot\beta|$ and $\langle e^{i\,\widetilde{\varphi}}\rangle = sign\,(A_t + \mu\cot\beta)$, but we do not make any assumption on their sign.

The two phases φ and $\widetilde{\varphi}$ depend on the Higgs background. Therefore, in the evaluation of the derivatives of $V_{\rm eff}$, their contribution should not be neglected. To simplify the calculations, we choose to redefine the fields in such a way that the top and stop mass matrices become real:

$$t_L' = e^{-i\frac{\varphi}{2}}t_L\,, \quad t_R' = e^{i\frac{\varphi}{2}}t_R\,, \qquad \tilde{t}_L' = e^{-i\frac{\widetilde{\varphi}}{2}}\tilde{t}_L\,, \quad \tilde{t}_R' = e^{i\frac{\widetilde{\varphi}}{2}}\tilde{t}_R\,. \tag{15}$$

This redefinition allows us to combine t_L and t_R, in the usual way, into a four–component Dirac spinor, with a real field–dependent mass $m_t \equiv h_t\,|H_2^0|$. Moreover, the field–dependent stop mass matrix is now real and symmetric, thus it can be diagonalized by the orthogonal transformation:

$$\left(\begin{array}{c}\tilde{t}_1\\ \tilde{t}_2\end{array}\right) = \left(\begin{array}{cc}\cos\bar{\theta}_{\tilde{t}} & \sin\bar{\theta}_{\tilde{t}}\\ -\sin\bar{\theta}_{\tilde{t}} & \cos\bar{\theta}_{\tilde{t}}\end{array}\right)\left(\begin{array}{c}\tilde{t}_L\\ \tilde{t}_R\end{array}\right)\,. \tag{16}$$

The field–dependent stop masses are:

$$m^2_{\tilde{t}_{1,2}} = \frac{1}{2}\left[(m_L^2 + m_R^2) \pm \sqrt{(m_L^2 - m_R^2)^2 + 4\,|\widetilde{X}|^2}\,\right]\,, \tag{17}$$

and the mixing angle $\bar{\theta}_{\tilde{t}}$ is also a field–dependent quantity, defined by:

$$\sin 2\bar{\theta}_{\tilde{t}} = \frac{2\,|\widetilde{X}|}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\,. \tag{18}$$

Notice that, in this case, $0 \leq \bar{\theta}_{\tilde{t}} < \pi/2$, in contrast with the usual field–independent definition for the angle $\theta_{\tilde{t}}$ that diagonalizes the stop mass matrix at the minimum,

$$\sin 2\theta_{\tilde{t}} = \frac{2\, m_t\, (A_t + \mu \cot\beta)}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\,, \tag{19}$$

which leads to $-\pi/2 \leq \theta_{\tilde{t}} < \pi/2$. The redefinition (15) has no effect on almost all field–dependent interaction vertices, with the only exception of the ones involving top, stop and gluino, which acquire a dependence on the phase difference $(\varphi - \widetilde{\varphi})$:

$$\mathcal{L}_{t\tilde{t}\tilde{g}} = -\sqrt{2}\, g_s \left(\overline{t_L}\, \tilde{g}\, T\, \tilde{t}_L\, e^{\frac{i}{2}(\varphi - \widetilde{\varphi})} - \overline{t_R}\, \tilde{g}\, T\, \tilde{t}_R\, e^{\frac{i}{2}(\widetilde{\varphi} - \varphi)} \right) + \text{h.c.}\,, \tag{20}$$

where T are the $SU(3)$ generators in the fundamental representation and all color indices are understood.

Before presenting the results for the $\mathcal{O}(\alpha_t \alpha_s)$ corrections to the neutral Higgs boson masses, we discuss the general structure of the pure SQCD corrections to the one–loop $\mathcal{O}(\alpha_t)$ results, namely the $\mathcal{O}(\alpha_t \alpha_s^n)$ terms for generic $n > 0$. For the computation of this class of corrections, the effective potential can be expressed as a function of five field–dependent quantities, which can be chosen as follows. The masses $m_t^2 = h_t^2 |H_2^0|^2$, $m^2_{\tilde{t}_1}$ and $m^2_{\tilde{t}_2}$, the last two as defined in eq. (17). The mixing parameter $c^2_{2\bar{\theta}} \equiv 1 - \sin^2 2\bar{\theta}_{\tilde{t}}$, where $\sin 2\bar{\theta}_{\tilde{t}}$ is given in eq. (18). Finally, a parameter that, according to eq. (20), should be a function of the phase difference $\varphi - \widetilde{\varphi}$: we conveniently choose it as $c_{\varphi\tilde{\varphi}} \equiv \cos(\varphi - \tilde{\varphi})$, where

$$\cos(\varphi - \tilde{\varphi}) = \frac{\mathrm{Re}(X)\, \mathrm{Re}(\widetilde{X}) + \mathrm{Im}(X)\, \mathrm{Im}(\widetilde{X})}{|X|\, |\widetilde{X}|}\,, \tag{21}$$

X and $\widetilde{X}$ are defined in eqs. (11) and (14), respectively, and $\langle c_{\varphi\tilde{\varphi}} \rangle = \pm 1$. A sixth parameter, the gluino mass $m_{\tilde{g}}$, does not depend on the Higgs background: we can restrict ourselves to positive values of $m_{\tilde{g}}$ if we neglect possible CP–violating phases and we allow for arbitrary signs of μ and A_t.

According to eq. (8), in the limit of neglecting the $SU(2)_L \times U(1)_Y$ gauge contributions beyond the tree level, the radiative corrections to the neutral CP–even Higgs boson mass matrix can be parametrized as:

$$\left(\mathcal{M}^2_S\right)_{ij} = \left(\mathcal{M}^2_S\right)^0_{ij} + \left(\Delta\mathcal{M}^2_S\right)_{ij}\,, \tag{22}$$

where

$$\left(\mathcal{M}^2_S\right)^0_{ij} = (-1)^{i+j} \left[\left(\mathcal{M}^2_P\right)_{ij} + \frac{(g^2 + g'^2)}{2} v_i v_j \right] \tag{23}$$

is fully determined by the input parameters m_Z, m_A and $\tan\beta$ since, at $\mathcal{O}(\alpha_s \alpha_t)$, $m_Z^2 = (g^2 + g'^2)(v_1^2 + v_2^2)/4$. The corrections that have not been reabsorbed in $(\mathcal{M}^2_S)^0$ are contained in:

$$\left(\Delta\mathcal{M}^2_S\right)_{ij} = -(-1)^{i+j} \left.\frac{\partial^2 V}{\partial P_i \partial P_j}\right|_{\text{min}} + \left.\frac{\partial^2 V}{\partial S_i \partial S_j}\right|_{\text{min}}\,. \tag{24}$$

Exploiting the field–dependence of the parameters, a wise although lengthy application of the chain rule for the derivatives of the effective potential leads to:

$$\left(\Delta\mathcal{M}_S^2\right)_{11} = \frac{1}{2}\, h_t^2\, \mu^2\, s_{2\theta}^2\, F_3\,, \tag{25}$$

$$\left(\Delta\mathcal{M}_S^2\right)_{12} = h_t^2\, \mu\, m_t\, s_{2\theta}\, F_2 + \frac{1}{2}\, h_t^2\, A_t\, \mu\, s_{2\theta}^2\, \left(F_3 + \Delta F_3\right)\,, \tag{26}$$

$$\left(\Delta\mathcal{M}_S^2\right)_{22} = 2\, h_t^2\, m_t^2\, F_1 + 2\, h_t^2\, A_t\, m_t\, s_{2\theta}\, \left(F_2 + \Delta F_2\right) + \frac{1}{2}\, h_t^2\, A_t^2\, s_{2\theta}^2\, \left(F_3 + 2\,\Delta F_3\right)\,, \tag{27}$$

where $s_{2\theta} \equiv \sin 2\theta_{\tilde{t}}$ refers to the stop mixing angle defined in the usual field–independent way [see eq. (19)]. The functions F_i $(i = 1, 2, 3)$ can be decomposed as $F_i = \widetilde{F}_i + \Delta\widetilde{F}_i$, where the $\Delta\widetilde{F}_i$ include the renormalization of the common factors multiplying F_i (i.e. h_t^2, m_t, $s_{2\theta}$), and

$$\begin{aligned}
\widetilde{F}_1 \;=\;& \frac{\partial^2 V}{(\partial m_t^2)^2} + \frac{\partial^2 V}{(\partial m_{\tilde{t}_1}^2)^2} + \frac{\partial^2 V}{(\partial m_{\tilde{t}_2}^2)^2} \\
& + 2\,\frac{\partial^2 V}{\partial m_t^2 \partial m_{\tilde{t}_1}^2} + 2\,\frac{\partial^2 V}{\partial m_t^2 \partial m_{\tilde{t}_2}^2} + 2\,\frac{\partial^2 V}{\partial m_{\tilde{t}_1}^2\, \partial m_{\tilde{t}_2}^2} + \frac{1}{4\, m_t^4}\, \frac{\partial V}{\partial c_{\varphi\tilde{\varphi}}}\,,
\end{aligned} \tag{28}$$

$$\begin{aligned}
\widetilde{F}_2 \;=\;& \frac{\partial^2 V}{(\partial m_{\tilde{t}_1}^2)^2} - \frac{\partial^2 V}{(\partial m_{\tilde{t}_2}^2)^2} + \frac{\partial^2 V}{\partial m_t^2 \partial m_{\tilde{t}_1}^2} - \frac{\partial^2 V}{\partial m_t^2 \partial m_{\tilde{t}_2}^2} - \frac{(s_{2\theta})^{-2}}{m_t^2\, (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)}\, \frac{\partial V}{\partial c_{\varphi\tilde{\varphi}}} \\
& - \frac{4\, c_{2\theta}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \left(\frac{\partial^2 V}{\partial c_{2\bar{\theta}}^2 \partial m_t^2} + \frac{\partial^2 V}{\partial c_{2\bar{\theta}}^2\, \partial m_{\tilde{t}_1}^2} + \frac{\partial^2 V}{\partial c_{2\bar{\theta}}^2\, \partial m_{\tilde{t}_2}^2} \right)\,,
\end{aligned} \tag{29}$$

$$\begin{aligned}
\widetilde{F}_3 \;=\;& \frac{\partial^2 V}{(\partial m_{\tilde{t}_1}^2)^2} + \frac{\partial^2 V}{(\partial m_{\tilde{t}_2}^2)^2} - 2\,\frac{\partial^2 V}{\partial m_{\tilde{t}_1}^2\, \partial m_{\tilde{t}_2}^2} - \frac{2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \left(\frac{\partial V}{\partial m_{\tilde{t}_1}^2} - \frac{\partial V}{\partial m_{\tilde{t}_2}^2} \right) + \frac{4\,(s_{2\theta})^{-4}}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2}\, \frac{\partial V}{\partial c_{\varphi\tilde{\varphi}}} \\
& + \frac{16\, c_{2\theta}^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2} \left(c_{2\theta}^2\, \frac{\partial^2 V}{(\partial c_{2\bar{\theta}}^2)^2} + 2\, \frac{\partial V}{\partial c_{2\bar{\theta}}^2} \right) - \frac{8\, c_{2\theta}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \left(\frac{\partial^2 V}{\partial c_{2\bar{\theta}}^2\, \partial m_{\tilde{t}_1}^2} - \frac{\partial^2 V}{\partial c_{2\bar{\theta}}^2\, \partial m_{\tilde{t}_2}^2} \right)\,.
\end{aligned} \tag{30}$$

In the above equations, $c_{2\theta}^2 = 1 - s_{2\theta}^2$ and the derivatives are evaluated at the minimum of $V_{\rm eff}$. As can be seen from eqs. (25)–(27), in every entry of $\Delta\mathcal{M}_S^2$ the F_i terms are multiplied by different combinations of μ and A_t. These two parameters do not renormalize in the same way, thus the contributions induced by their renormalization cannot be absorbed into the F_i, but should be separately taken into account. However, since at $\mathcal{O}(\alpha_s)$ μ does not renormalize, we have inserted in eqs. (26)–(27) only the two factors ΔF_2 and ΔF_3 that take into account the renormalization of A_t.

To evaluate the functions (F_1, F_2, F_3), two strategies come to mind: i) evaluate explicitly the effective potential and then differentiate with respect to the relevant field–dependent quantities; ii) use a well–known fact, that the derivative of a bubble diagram with respect to an internal mass is still a diagram of the same kind, to compute directly the derivatives of the effective potential, without evaluating the effective potential itself. In our calculation of the $\mathcal{O}(\alpha_t\alpha_s)$ corrections to the neutral Higgs boson masses we followed the latter strategy. The corresponding corrections to the effective potential can be found in eq. (4) of the second paper of ref. [15], with the understanding that the last two terms, proportional to $m_{\tilde{g}} m_t s_{2\theta}$, should be multiplied by $c_{\varphi\tilde{\varphi}}$. However, only the second derivatives of $c_{\varphi\tilde{\varphi}}$ with respect to the fields P_i are different from zero at the minimum of the potential. Therefore, this extra term does not contribute to the expression for m_h in the decoupling limit of very large m_A, where the results of that paper are applicable.

We give now the explicit expressions for the $\mathcal{O}(\alpha_t\alpha_s)$ contribution to the functions (F_1, F_2, F_3). For completeness, we recall first the one–loop result [6]:

$$F_1^{1\ell} = \frac{N_c}{16\,\pi^2}\ln\frac{m_{\tilde t_1}^2\,m_{\tilde t_2}^2}{m_t^4}\,,\qquad F_2^{1\ell} = \frac{N_c}{16\,\pi^2}\ln\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\,,\qquad F_3^{1\ell} = \frac{N_c}{16\,\pi^2}\left(2-\frac{m_{\tilde t_1}^2+m_{\tilde t_2}^2}{m_{\tilde t_1}^2-m_{\tilde t_2}^2}\ln\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\right)\,, \tag{31}$$

where $N_c = 3$ is a color factor. We assume that the $\mathcal{O}(\alpha_t)$ one–loop contribution is written in terms of top and stop parameters evaluated in the $\overline{\rm DR}$ scheme [v_1 and v_2 are automatically defined in the $\overline{\rm DR}$ scheme by eqs. (4) and (5), and the same is true for $\tan\beta = v_2/v_1$]. The two–loop $\mathcal{O}(\alpha_t\alpha_s)$ contributions to the functions (F_1, F_2, F_3), in units of $g_s^2\,C_F\,N_c/(16\,\pi^2)^2$ (where $C_F = 4/3$), and in the $\overline{\rm DR}$ renormalization scheme (here and hereafter $\overline{\rm DR}$ quantities will be denoted by a hat), are given by:

$$\begin{aligned}\hat F_1^{2\ell} \;=\;& -6\left(1-\ln\frac{m_t^2}{Q^2}\right)+5\,\ln\frac{m_{\tilde t_1}^2\,m_{\tilde t_2}^2}{m_t^4}+\ln^2\frac{m_{\tilde t_1}^2\,m_{\tilde t_2}^2}{m_t^4}+8\,\ln^2\frac{m_t^2}{Q^2}\\ &-4\left(\ln^2\frac{m_{\tilde t_1}^2}{Q^2}+\ln^2\frac{m_{\tilde t_2}^2}{Q^2}\right)-\,c_{2\theta}^2\left[2-\ln\frac{m_{\tilde t_1}^2\,m_{\tilde t_2}^2}{Q^4}-\ln^2\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\right]\\ &-\,s_{2\theta}^2\left[\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\left(1-\ln\frac{m_{\tilde t_1}^2}{Q^2}\right)+\frac{m_{\tilde t_2}^2}{m_{\tilde t_1}^2}\left(1-\ln\frac{m_{\tilde t_2}^2}{Q^2}\right)\right]\\ &+\,f_1(m_t\,,m_{\tilde g}\,,m_{\tilde t_1}^2\,,m_{\tilde t_2}^2\,,s_{2\theta}\,,Q)\;+\;f_1(m_t\,,m_{\tilde g}\,,m_{\tilde t_2}^2\,,m_{\tilde t_1}^2\,,-s_{2\theta}\,,Q)\;,\end{aligned} \tag{32}$$

$$\begin{aligned}\hat F_2^{2\ell} \;=\;& 5\,\ln\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}-3\left(\ln^2\frac{m_{\tilde t_1}^2}{Q^2}-\ln^2\frac{m_{\tilde t_2}^2}{Q^2}\right)+\,c_{2\theta}^2\left[5\,\ln\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\right.\\ &\left.-\frac{m_{\tilde t_1}^2+m_{\tilde t_2}^2}{m_{\tilde t_1}^2-m_{\tilde t_2}^2}\ln^2\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}-\frac{2}{m_{\tilde t_1}^2-m_{\tilde t_2}^2}\left(m_{\tilde t_1}^2\,\ln\frac{m_{\tilde t_1}^2}{Q^2}-m_{\tilde t_2}^2\,\ln\frac{m_{\tilde t_2}^2}{Q^2}\right)\ln\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\right]\\ &+\,s_{2\theta}^2\left[\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\left(1-\ln\frac{m_{\tilde t_1}^2}{Q^2}\right)-\frac{m_{\tilde t_2}^2}{m_{\tilde t_1}^2}\left(1-\ln\frac{m_{\tilde t_2}^2}{Q^2}\right)\right]\\ &+\,f_2(m_t\,,m_{\tilde g}\,,m_{\tilde t_1}^2\,,m_{\tilde t_2}^2\,,s_{2\theta}\,,Q)\;-\;f_2(m_t\,,m_{\tilde g}\,,m_{\tilde t_2}^2\,,m_{\tilde t_1}^2\,,-s_{2\theta}\,,Q)\;,\end{aligned} \tag{33}$$

$$\begin{aligned}\hat F_3^{2\ell} \;=\;& \frac{16\pi^2}{N_c}F_3^{1\ell}\,(3+9\,c_{2\theta}^2)+4-\frac{3+13\,c_{2\theta}^2}{m_{\tilde t_1}^2-m_{\tilde t_2}^2}\left(m_{\tilde t_1}^2\,\ln\frac{m_{\tilde t_1}^2}{Q^2}-m_{\tilde t_2}^2\,\ln\frac{m_{\tilde t_2}^2}{Q^2}\right)\\ &+3\,\frac{m_{\tilde t_1}^2+m_{\tilde t_2}^2}{m_{\tilde t_1}^2-m_{\tilde t_2}^2}\left(\ln^2\frac{m_{\tilde t_1}^2}{Q^2}-\ln^2\frac{m_{\tilde t_2}^2}{Q^2}\right)-\,c_{2\theta}^2\left[4-\left(\frac{m_{\tilde t_1}^2+m_{\tilde t_2}^2}{m_{\tilde t_1}^2-m_{\tilde t_2}^2}\right)^2\ln^2\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\right.\\ &\left.-6\,\frac{m_{\tilde t_1}^2+m_{\tilde t_2}^2}{(m_{\tilde t_1}^2-m_{\tilde t_2}^2)^2}\left(m_{\tilde t_1}^2\,\ln\frac{m_{\tilde t_1}^2}{Q^2}-m_{\tilde t_2}^2\,\ln\frac{m_{\tilde t_2}^2}{Q^2}\right)\ln\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}\right]-\,s_{2\theta}^2\left[\frac{m_{\tilde t_1}^2}{m_{\tilde t_2}^2}+\frac{m_{\tilde t_2}^2}{m_{\tilde t_1}^2}\right.\end{aligned}$$

$$+2\,\ln\frac{m^2_{\tilde t_1}\,m^2_{\tilde t_2}}{Q^4} - \frac{m^4_{\tilde t_1}}{m^2_{\tilde t_2}\,(m^2_{\tilde t_1}-m^2_{\tilde t_2})}\,\ln\frac{m^2_{\tilde t_1}}{Q^2} + \frac{m^4_{\tilde t_2}}{m^2_{\tilde t_1}\,(m^2_{\tilde t_1}-m^2_{\tilde t_2})}\,\ln\frac{m^2_{\tilde t_2}}{Q^2}\,\Bigg]$$

$$+\; f_3(m_t\,,m_{\tilde g}\,,m^2_{\tilde t_1}\,,m^2_{\tilde t_2}\,,s_{2\theta}\,,Q) \;+\; f_3(m_t\,,m_{\tilde g}\,,m^2_{\tilde t_2}\,,m^2_{\tilde t_1}\,,-s_{2\theta}\,,Q)\;. \tag{34}$$

In the above expressions, Q indicates the $\overline{\rm DR}$ renormalization scale, and the functions f_i contain contributions coming from the top-stop-gluino diagrams (fig. 1d): their explicit expressions are presented in appendix A.

To obtain the $\mathcal{O}(\alpha_t\alpha_s)$ corrections to the Higgs mass entries, we also need explicit expressions for the ΔF_i terms of eqs. (26)–(27). In the $\overline{\rm DR}$ scheme, and in units of $g_s^2\,C_F\,N_c/(16\,\pi^2)^2$, they are:

$$\Delta\hat F_2 \;=\; \frac{2\,m_{\tilde g}}{A_t}\left(\ln^2\frac{m^2_{\tilde t_2}}{Q^2} - \ln^2\frac{m^2_{\tilde t_1}}{Q^2}\right)\,, \tag{35}$$

$$\begin{aligned}\Delta\hat F_3 \;=\; & \frac{m_{\tilde g}}{A_t}\left[8 - 2\,\frac{m^2_{\tilde t_1}+m^2_{\tilde t_2}}{m^2_{\tilde t_1}-m^2_{\tilde t_2}}\left(\ln^2\frac{m^2_{\tilde t_2}}{Q^2} - \ln^2\frac{m^2_{\tilde t_1}}{Q^2}\right)\right.\\ & \left. + \frac{8}{m^2_{\tilde t_1}-m^2_{\tilde t_2}}\left(m^2_{\tilde t_2}\,\ln\frac{m^2_{\tilde t_2}}{Q^2} - m^2_{\tilde t_1}\,\ln\frac{m^2_{\tilde t_1}}{Q^2}\right)\right]\,. \end{aligned}\tag{36}$$

A comment on eqs. (32)–(36) is in order. These equations show an explicit dependence on the renormalization scale Q, connected with our choice of expressing the top and stop parameters in the $\overline{\rm DR}$ scheme. This dependence is cancelled by the implicit dependence of the $\overline{\rm DR}$ parameters, so that the entries of $\Delta\mathcal{M}^2_S$ are scale–independent. This fact becomes manifest if we reexpress the top and stop parameters in a physical scheme such as the on–shell (OS) scheme. To ensure this scale–independence, it was crucial to include the contributions induced by $\partial V/\partial c_{\varphi\tilde\varphi}$. If these terms were neglected, and the limit $m_A\to\infty$ were taken, one would still find a scale–independent result for the $\mathcal{O}(\alpha_t\,\alpha_s)$ corrections to m_h (thanks to the fact that, in such a limit, m_h does not depend upon m_A), but not for the corrections to m_H. It may be useful to recall that, because v_1 and v_2 are automatically defined in the $\overline{\rm DR}$ scheme, $\left(\mathcal{M}^2_S\right)^0$ has an implicit scale dependence, since $\mathcal{M}^2_P$ in eq. (23) contains $\tan\beta$. This residual scale dependence could be removed by including the momentum–dependent parts of the self–energies in the two–loop computation.

To obtain the $\mathcal{O}(\alpha_t\alpha_s)$ corrections in some other renormalization scheme, R, we just need to shift the top and stop parameters appearing in the one–loop term. Indicating the general mass in the $\overline{\rm DR}$ scheme as $m^{\overline{\rm DR}}$, and the same quantity in the R scheme as m, we can write the one–loop relation $m = m^{\overline{\rm DR}} - \delta m$. Then, once the one–loop contribution is written in terms of R quantities, the two–loop $\mathcal{O}(\alpha_t\alpha_s)$ corrections in the R scheme can be obtained through:

$$F_1^{2\ell} \;=\; \hat F_1^{2\ell} + \frac{N_c}{16\,\pi^2}\left(\frac{\delta m^2_{\tilde t_1}}{m^2_{\tilde t_1}} + \frac{\delta m^2_{\tilde t_2}}{m^2_{\tilde t_2}} - 4\,\frac{\delta m_t}{m_t}\right) + 4\,\frac{\delta m_t}{m_t}\,F_1^{1\ell} \tag{37}$$

$$F_2^{2\ell} \;=\; \hat F_2^{2\ell} + \frac{N_c}{16\,\pi^2}\left(\frac{\delta m^2_{\tilde t_1}}{m^2_{\tilde t_1}} - \frac{\delta m^2_{\tilde t_2}}{m^2_{\tilde t_2}}\right) + \left(3\,\frac{\delta m_t}{m_t} + \frac{\delta s_{2\theta}}{s_{2\theta}}\right)\,F_2^{1\ell}\,, \tag{38}$$

$$
\begin{aligned}
F_3^{2\ell} &= \hat{F}_3^{2\ell} + \frac{N_c}{16\,\pi^2}\left(2\,\frac{m_{\tilde{t}_1}^2\, m_{\tilde{t}_2}^2}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2}\ln\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} - \frac{m_{\tilde{t}_1}^2 + m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2}\right)\left(\frac{\delta m_{\tilde{t}_1}^2}{m_{\tilde{t}_1}^2} - \frac{\delta m_{\tilde{t}_2}^2}{m_{\tilde{t}_2}^2}\right) \\
&\quad + \left(2\,\frac{\delta m_t}{m_t} + 2\,\frac{\delta s_{2\theta}}{s_{2\theta}}\right) F_3^{1\ell}\,, \qquad (39)
\end{aligned}
$$

$$
(\Delta F_2) = (\Delta \hat{F}_2) + \frac{\delta A_t}{A_t}\, F_2^{1\ell}\,, \qquad (40)
$$

$$
(\Delta F_3) = (\Delta \hat{F}_3) + \frac{\delta A_t}{A_t}\, F_3^{1\ell}\,, \qquad (41)
$$

where all the quantities that appear in eqs. (37–41) are meant in the R scheme. The explicit expressions that allow us to perform the one–loop shift for (m_t, $m_{\tilde{t}_1}^2$, $m_{\tilde{t}_2}^2$, $s_{2\theta}$, A_t), from the $\overline{\rm DR}$ to the OS renormalization scheme, are listed in appendix B.

4 Discussion

Using the formalism of the effective potential, we have obtained complete, explicit, analytical expressions for the $\mathcal{O}(\alpha_t\alpha_s)$ two–loop corrections to the MSSM mass matrix for the CP–even Higgs bosons. Our input parameters are: (m_Z, m_A, $\tan\beta$), already appearing in the tree-level result; the parameters of the top and stop sectors, appearing in the $\mathcal{O}(\alpha_t)$ one–loop correction, for example (m_t, $m_{\tilde{t}_1}^2$, $m_{\tilde{t}_2}^2$, $s_{2\theta}$, μ); the gluino mass and the strong coupling constant, appearing only at the two–loop level. We have presented our results in such a way that the input parameters of the top and stop sectors can be given either in the $\overline{\rm DR}$ scheme or in some version of the on–shell scheme. Also, we have included in appendix C the $\mathcal{O}(\alpha_t\alpha_s)$ corrections to the relation that gives m_A^2 in terms of m_3^2 and $\tan\beta$. This result can be useful if one deals with models that predict the low–energy values of the soft supersymmetry–breaking parameters.

Our effective potential calculation is equivalent to the evaluation of the Higgs self–energies in the limit of vanishing external momentum. A diagrammatic computation of the two–loop $\mathcal{O}(\alpha_t\alpha_s)$ contributions to the Higgs boson self–energies at zero external momentum has been performed in [13]. Analytical formulae, valid in the simplified case of degenerate soft stop masses and zero mixing (with $\mu = A_t = 0$), have been presented in the first paper of ref. [13]. For arbitrary values of the top and stop parameters, however, the complete analytical result of [13] is far too long to be explicitly presented, and is only available as a computer code [18]. We have checked that, in the case of zero mixing and degenerate stop masses, our results coincide with those of [13]. Moreover, after taking into account the difference in the definitions of the on–shell renormalized angle $\theta_{\tilde{t}}$ (see appendix B), we find perfect agreement with the numerical results of [18], for arbitrary values of all the input parameters.

A calculation of both $\mathcal{O}(\alpha_t\,\alpha_s)$ and $\mathcal{O}(\alpha_t^2)$ two–loop corrections to the lightest Higgs boson mass m_h, based on the formalism of the effective potential, has been presented in [15]. In these papers, however, the dependence of the stop masses and mixing angles on the fields P_i (the CP–odd components of the neutral Higgs fields) is not taken into consideration. Therefore, the $\mathcal{O}(\alpha_t\,\alpha_s)$ corrections to the input parameter m_A are not evaluated. If one wants to relate the input parameters to measurable quantities, the computation is applicable only in the limit $m_A \gg m_Z$, in which m_h is nearly independent of m_A. However, the results of [15] allow to express m_h as a function of the (unphysical) renormalized parameter m_3^2 in the $\overline{\rm DR}$ scheme.

Moreover, while an analytical formula for $V_{\rm eff}$ is given (in terms of m_3^2 and of the fields S_i), the results of [15] for m_h are available in numerical form, and simple analytical formulae are provided only in the case of universal soft stop masses and small stop mixing.

The corrections controlled by α_s introduce a new mass scale in the prediction of the MSSM neutral Higgs boson masses, namely the gluino mass $m_{\tilde{g}}$. To avoid dealing with many different scales, $m_{\tilde{g}}$ is usually set to be of the same magnitude of the stop masses. Notice that, in a scenario where stops and gluinos are all heavy and approximately degenerate, with masses $\mathcal{O}(M_S)$, only the function $\hat{F}_1$ contains large logarithms of the ratio m_t/M_S. Instead, as we have explicitly checked, $\hat{F}_2$ and $\hat{F}_3$ are finite in the limit $m_t \to 0$. They contribute only to the matching conditions between the MSSM and the effective theory below the scale M_S, to be identified at $\mathcal{O}(\alpha_t\alpha_s)$ with a two–Higgs–doublet version of the Standard Model. However, if the term $A_t + \mu \cot\beta$ is very large, or m_L and m_R are very different, or both, the two stop mass eigenstates can have a wide mass gap, and large logarithms of the ratio $m_{\tilde{t}_1}/m_{\tilde{t}_2}$ can then be present in all the $\hat{F}_i$ terms. It should be mentioned that in this case the low-energy effective theory is different from a two–Higgs–doublet version of the SM, and indeed much more complicated already in the case of small stop mixing [9], not to mention the difficult case of large stop mixing.

We can also envisage a scenario in which the gluino is much heavier than the top and the stops. Eqs. (A1)–(A3) contain terms proportional to powers of $m_{\tilde{g}}$ that can be potentially large. This powerlike behavior is actually cancelled in the OS schemes by the finite parts of the relevant shifts. However, as already noticed in [13], the gluino does not fully decouple, and m_h increases logarithmically with $m_{\tilde{g}}$ when the latter becomes very large. On the other hand, it must be noticed that, in the $\overline{\rm DR}$ scheme, some terms proportional to $m_{\tilde{g}}$ and $m_{\tilde{g}}^2$ are not cancelled, and in the limit of heavy gluino the two–loop corrections to the Higgs masses can become very large : this is related with the non–decoupling properties of mass–independent renormalization schemes such as $\overline{\rm DR}$.

From eqs. (A1)–(A3), we can derive approximate formulae for the two–loop corrections to $\mathcal{M}_S^2$ in the case of large gluino mass, keeping the leading terms in an expansion of the complete result in powers of $m_{\tilde{g}}$. Doing so, the asymptotic behavior is approached quite slowly as $m_{\tilde{g}}$ increases. The reason is that some terms that are formally suppressed by inverse powers of $m_{\tilde{g}}$ are indeed enhanced by large numerical factors. In order to get an accurate approximation to the correct result, it is then preferable to include also the next–to–leading terms in the expansion in powers of $m_{\tilde{g}}$. Specializing for simplicity to the case of degenerate soft stop masses and $\mu = A_t = 0$, so that $m^2_{\tilde{t}_1} = m^2_{\tilde{t}_2} \equiv m^2_{\tilde{t}}$ and the only non–zero correction to the Higgs mass matrix is $(\Delta\mathcal{M}_S^2)_{22}$, we find:

$$
\begin{aligned}
\left(\Delta\mathcal{M}_S^2\right)_{22} \;&=\; \frac{h_t^2\, g_s^2}{8\,\pi^4}\, m_t^2 \left(\frac{2\,\pi^2}{3} - 1 - 6\,\ln\frac{m_{\tilde{g}}^2}{m_t^2} - 3\,\ln^2\frac{m_{\tilde{t}}^2}{m_t^2} + 2\,\ln^2\frac{m_{\tilde{g}}^2}{m_{\tilde{t}}^2} \right) \\
&+ \frac{h_t^2\, g_s^2}{64\,\pi^4}\, \frac{m_t^2}{m_{\tilde{g}}^2} \left(\frac{32\,\pi^2}{3}\,(2\,m_t^2 + m_{\tilde{t}}^2) - \frac{160\,m_t^2 + 112\,m_{\tilde{t}}^2}{3} - \frac{352}{3}\, m_t^2\, \ln\frac{m_{\tilde{g}}^2}{m_t^2} + 32\,m_t^2\,\ln^2\frac{m_{\tilde{g}}^2}{m_t^2} \right. \\
&\left. - \frac{224\,m_t^2 + 288\,m_{\tilde{t}}^2}{3}\, \ln\frac{m_{\tilde{g}}^2}{m_{\tilde{t}}^2} + 32\,(m_t^2 + m_{\tilde{t}}^2)\,\ln^2\frac{m_{\tilde{g}}^2}{m_{\tilde{t}}^2} - 32\,m_t^2\,\ln^2\frac{m_{\tilde{t}}^2}{m_t^2} \right) \;+\; \mathcal{O}\left(m_{\tilde{g}}^{-4}\right)\,. \qquad (42)
\end{aligned}
$$

In deriving eq. (42), we have assumed an on–shell renormalization for the top and stop parameters. As anticipated above, if we were to write the one–loop corrections in terms of $\overline{\rm DR}$

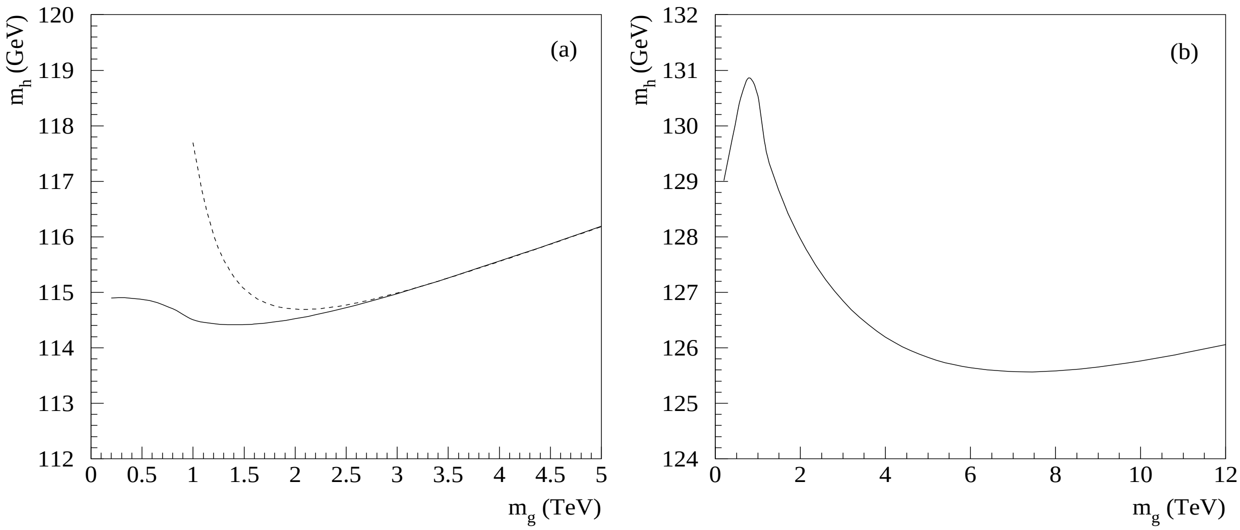

Figure 2: The mass m_h from our effective potential calculation, as a function of the gluino mass $m_{\tilde{g}}$, for (a) no mixing ($m_{\tilde{t}_1} = m_{\tilde{t}_2} = 1015$ GeV, $s_{2\theta} = 0$) and (b) large mixing ($m_{\tilde{t}_1} = 1175$ GeV, $m_{\tilde{t}_2} = 825$ GeV, $s_{2\theta} = 1$) in the stop sector. The other MSSM parameters are: $m_A = 500$ GeV, $\tan\beta = 10$, $\mu = 0$ and $m_t = 175$ GeV. The dashed line in fig. 2a is the approximate result of eq. (42).

parameters, we would find in $\Delta\mathcal{M}_S^2$ terms proportional to $m_{\tilde{g}}^2$. In fact, as can be seen from eq. (B3), the finite shift $\delta m_{\tilde{t}}^2$ scales for large $m_{\tilde{g}}$ as $m_{\tilde{g}}^2\,(\ln(m_{\tilde{g}}^2/Q^2) - 1)$, and cancels in the OS scheme similar terms present in the f_i.

Some representative results for the heavy–gluino limit are shown in fig. 2. We have plotted m_h, as obtained from our complete formulae, as a function of the gluino mass. We employ on–shell top and stop parameters (for their definitions, see appendix B). We consider two examples, with degenerate soft stop masses and either no mixing or large mixing in the stop sector. The numerical inputs we use are: $m_A = 500$ GeV, $\tan\beta = 10$, $\mu = 0$, $m_t = 175$ GeV, and the values $m_{\tilde{t}_1} = m_{\tilde{t}_2} = 1015$ GeV, $s_{2\theta} = 0$ (no mixing, fig. 2a) and $m_{\tilde{t}_1} = 1175$ GeV, $m_{\tilde{t}_2} = 825$ GeV, $s_{2\theta} = 1$ (large mixing, fig. 2b). For the one–loop $\mathcal{O}(\alpha_t)$ corrections to $\mathcal{M}_S^2$ we have used the effective potential result of [6]. As can be seen from figure 2, m_h reaches a maximum at low values of $m_{\tilde{g}}$, decreases for intermediate values and then increases logarithmically when $m_{\tilde{g}}$ becomes very large. Comparing the cases of no mixing and large stop mixing, we see that in the latter case the peak of m_h at small $m_{\tilde{g}}$ is much more pronounced, and the asymptotic increase of m_h starts at higher values of $m_{\tilde{g}}$. In fig. 2a we have also plotted m_h as obtained with the approximate formula of eq. (42), valid in the case of no mixing. One can see that eq. (42) approximates very well the complete result when $m_{\tilde{g}} > 2\,m_{\tilde{t}}$.

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Appendix A: Analytical expressions for the gluino contributions

The explicit expressions of the contributions to the functions $\hat{F}_i^{2\ell}$ $(i = 1, 2, 3)$ coming from the top–stop–gluino diagrams (fig. 1d) are rather long but, as apparent from eqs. (32)–(34), they possess useful symmetry properties under the exchanges $m^2_{\tilde{t}_1} \leftrightarrow m^2_{\tilde{t}_2}$, $s_{2\theta} \leftrightarrow -s_{2\theta}$. In units of $g_s^2\, C_F\, N_c/(16\,\pi^2)^2$, where $C_F = 4/3$ and $N_c = 3$ are color factors, the explicit expressions of the functions f_i $(i = 1, 2, 3)$ are:

$$
\begin{aligned}
f_1(m_t\,, m_{\tilde g}\,, m^2_{\tilde t_1}\,, m^2_{\tilde t_2}\,, s_{2\theta}\,, Q) \;&=\; 4\,\frac{m_t^2 + m_{\tilde g}^2 - \, m_{\tilde g}\, m_t\, s_{2\theta}}{m^2_{\tilde t_1}} \left(1 - \ln\frac{m^2_{\tilde g}}{Q^2}\right) + 4\,\ln\frac{m_t^2}{m_{\tilde g}^2} \\
&-2\,\ln\frac{m^2_{\tilde t_1}}{m^2_{\tilde g}} \;+\; \frac{2}{\Delta}\left[4\,m_{\tilde g}^4\,\ln\frac{m^2_{\tilde t_1}}{m^2_{\tilde g}} + \left(m_{\tilde g}^4 - m^4_{\tilde t_1} + m_t^2\left(10\,m_{\tilde g}^2 + 3\,m_t^2 + 2\,\frac{m_t^2\,m_{\tilde g}^2 - m_t^4}{m^2_{\tilde t_1}}\right)\right)\ln\frac{m_t^2}{m_{\tilde g}^2}\right] \\
&+\frac{2\,m_{\tilde g}\,s_{2\theta}}{m_t}\left(\ln^2\frac{m^2_{\tilde t_1}}{Q^2} + 2\,\ln\frac{m_t^2}{Q^2}\,\ln\frac{m^2_{\tilde t_1}}{Q^2}\right) + \frac{4\,m_{\tilde g}\,s_{2\theta}}{m_t\,\Delta}\left[m_{\tilde g}^2\,\left(m^2_{\tilde t_1} - m_t^2 - m_{\tilde g}^2\right)\,\ln\frac{m^2_{\tilde t_1}}{m^2_{\tilde g}}\right. \\
&\qquad\left. + \,m_t^2\left(m^2_{\tilde t_1} - 3\,m_{\tilde g}^2 - 2\,m_t^2 - \frac{m_t^2\,m_{\tilde g}^2 - m_t^4}{m^2_{\tilde t_1}}\right)\ln\frac{m_t^2}{m_{\tilde g}^2}\right] \\
&+\left[\frac{4\,m_{\tilde g}^2\,\left(m_t^2 + m_{\tilde g}^2 - m^2_{\tilde t_1} - 2\,m_{\tilde g}\,m_t\,s_{2\theta}\right)}{\Delta} - \frac{4\,m_{\tilde g}\,s_{2\theta}}{m_t}\right]\Phi\,(m_t^2, m^2_{\tilde t_1}, m_{\tilde g}^2)\,, \qquad \text{(A1)}
\end{aligned}
$$

$$
\begin{aligned}
f_2(m_t\,, m_{\tilde g}\,, m^2_{\tilde t_1}\,, m^2_{\tilde t_2}\,, s_{2\theta}\,, Q) \;&=\; 4\,\frac{m_t^2 + m_{\tilde g}^2}{m^2_{\tilde t_1}} - \frac{4\,m_{\tilde g}\,s_{2\theta}}{m_t\,(m^2_{\tilde t_1} - m^2_{\tilde t_2})}\left(3\,m^2_{\tilde t_1} - \frac{m_t^2\,m^2_{\tilde t_2}}{m^2_{\tilde t_1}}\right) \\
&+\frac{2\,m_{\tilde g}\,s_{2\theta}}{m_t\,(m^2_{\tilde t_1} - m^2_{\tilde t_2})}\left[\left(4\,m_t^2 + 5\,m^2_{\tilde t_1} + m^2_{\tilde t_2}\right)\,\ln\frac{m^2_{\tilde t_1}}{Q^2} - 2\,\frac{m_t^2\,m^2_{\tilde t_2}}{m^2_{\tilde t_1}}\,\ln\frac{m_{\tilde g}^2}{Q^2}\right] \\
&-4\,\frac{m_{\tilde g}^2 + m_t^2}{m^2_{\tilde t_1}}\,\ln\frac{m_{\tilde g}^2}{Q^2} - 2\,\ln\frac{m^2_{\tilde t_1}}{m_{\tilde g}^2} + \frac{2}{\Delta}\left[2\,m_{\tilde g}^2\,\left(m_{\tilde g}^2 + m_t^2 - m^2_{\tilde t_1}\right)\,\ln\frac{m^2_{\tilde t_1}}{m_{\tilde g}^2}\right. \\
&\left. + \,2\,m_t^2\left(3\,m_{\tilde g}^2 + 2\,m_t^2 - m^2_{\tilde t_1} + \frac{m_{\tilde g}^2\,m_t^2 - m_t^4}{m^2_{\tilde t_1}}\right)\ln\frac{m_t^2}{m_{\tilde g}^2}\right] - \frac{4\,m_{\tilde g}\,m_t\,s_{2\theta}}{m^2_{\tilde t_1}\,\Delta}\left[2\,m^2_{\tilde t_1}\,m^2_{\tilde g}\,\ln\frac{m^2_{\tilde t_1}}{m_{\tilde g}^2}\right.
\end{aligned}
$$

$$
- \Big((m_t^2 - m_{\tilde t_1}^2)^2 - m_{\tilde g}^2\,(m_t^2 + m_{\tilde t_1}^2) \Big)\, \ln \frac{m_t^2}{m_{\tilde g}^2} \Bigg] - \frac{8\, m_{\tilde g}\, m_t}{s_{2\theta}\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)} \left[\ln \frac{m_{\tilde t_1}^2}{Q^2} - \ln \frac{m_t^2}{Q^2} \ln \frac{m_{\tilde t_1}^2}{Q^2} \right]
$$

$$
- \frac{m_{\tilde g}\, s_{2\theta}}{m_t\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)} \Bigg[\Big(m_{\tilde t_1}^2 + m_{\tilde t_2}^2 \Big) \ln^2 \frac{m_{\tilde t_1}^2}{Q^2} + \Big(10\, m_t^2 - 2\, m_{\tilde g}^2 + m_{\tilde t_1}^2 + m_{\tilde t_2}^2 \Big) \ln \frac{m_t^2}{Q^2} \ln \frac{m_{\tilde t_1}^2}{Q^2}
$$

$$
+ \Big(2\, m_{\tilde g}^2 - 2\, m_t^2 + m_{\tilde t_1}^2 + m_{\tilde t_2}^2 \Big) \ln \frac{m_{\tilde t_1}^2}{Q^2} \ln \frac{m_{\tilde g}^2}{Q^2} \Bigg] + \Bigg[\frac{8\, m_{\tilde g}^2\, m_t^2}{\Delta} - \frac{8\, m_{\tilde g}\, m_t}{s_{2\theta}\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}
$$

$$
+ \frac{2\, s_{2\theta}\,(4\, m_t^2\, m_{\tilde g}^2 - \Delta)}{m_{\tilde g}\, m_t\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)} + \frac{s_{2\theta}\,(m_{\tilde t_1}^2 - m_{\tilde g}^2 - m_t^2)^3}{m_{\tilde g}\, m_t\, \Delta} \Bigg] \, \Phi\,(m_t^2, m_{\tilde t_1}^2, m_{\tilde g}^2)\,, \tag{A2}
$$

$$
f_3(m_t\,, m_{\tilde g}\,, m_{\tilde t_1}^2\,, m_{\tilde t_2}^2\,, s_{2\theta}\,, Q) \;=\; -4\, \frac{m_{\tilde t_2}^2\,(m_{\tilde g}^2 + m_t^2)}{m_{\tilde t_1}^2\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)} + \frac{4\, m_{\tilde g}\, m_t\, s_{2\theta}}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)^2} \left(21\, m_{\tilde t_1}^2 - \frac{m_{\tilde t_2}^4}{m_{\tilde t_1}^2} \right)
$$

$$
+ \frac{4}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2} \left[\frac{m_{\tilde g}^2 m_{\tilde t_2}^2}{m_{\tilde t_1}^2} \ln \frac{m_{\tilde g}^2}{Q^2} - 2\,(m_t^2 + m_{\tilde g}^2) \ln \frac{m_{\tilde t_1}^2}{Q^2} \right] - \frac{24\, m_{\tilde g}\, m_t\, s_{2\theta}\,(3\, m_{\tilde t_1}^2 + m_{\tilde t_2}^2)}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)^2} \ln \frac{m_{\tilde t_1}^2}{Q^2}
$$

$$
+ \frac{4\, m_t^2}{m_{\tilde t_1}^2\, \Delta} \Bigg[2\, m_{\tilde g}^2\, m_{\tilde t_1}^2 \ln \frac{m_{\tilde t_1}^2}{Q^2} - m_{\tilde g}^2\,(m_{\tilde g}^2 - m_t^2 + m_{\tilde t_1}^2) \ln \frac{m_{\tilde g}^2}{Q^2}
$$

$$
- \Big((m_t^2 - m_{\tilde t_1}^2)^2 - m_{\tilde g}^2\,(m_t^2 + m_{\tilde t_1}^2) \Big) \ln \frac{m_t^2}{Q^2} \Bigg] - \frac{4\, m_{\tilde g}\, m_t\, s_{2\theta}}{m_{\tilde t_1}^2\, \Delta} \Bigg[m_t^2\,(m_{\tilde g}^2 - m_t^2 + m_{\tilde t_1}^2) \ln \frac{m_t^2}{Q^2}
$$

$$
- m_{\tilde g}^2\,(m_{\tilde g}^2 - m_t^2 - m_{\tilde t_1}^2) \ln \frac{m_{\tilde g}^2}{Q^2} + m_{\tilde t_1}^2\,(m_{\tilde g}^2 + m_t^2 - m_{\tilde t_1}^2) \ln \frac{m_{\tilde t_1}^2}{Q^2} \Bigg]
$$

$$
+ 2\, \frac{2\, m_{\tilde g}^2 + 2\, m_t^2 - m_{\tilde t_1}^2 - m_{\tilde t_2}^2}{m_{\tilde t_1}^2 - m_{\tilde t_2}^2} \ln \frac{m_t^2\, m_{\tilde g}^2}{Q^4} \ln \frac{m_{\tilde t_1}^2}{Q^2} + \frac{12\, m_{\tilde g}\, m_t\, s_{2\theta}}{(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)^2} \Bigg[2\,(m_{\tilde g}^2 - m_t^2) \ln \frac{m_{\tilde g}^2}{m_t^2} \ln \frac{m_{\tilde t_1}^2}{Q^2}
$$

$$
+ (m_{\tilde t_1}^2 + m_{\tilde t_2}^2) \ln \frac{m_t^2\, m_{\tilde g}^2}{Q^4} \ln \frac{m_{\tilde t_1}^2}{Q^2} \Bigg] + \frac{8\, m_{\tilde g}\, m_t}{s_{2\theta}\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)^2} \Bigg[-8\, m_{\tilde t_1}^2 + 2\,(3\, m_{\tilde t_1}^2 + m_{\tilde t_2}^2) \ln \frac{m_{\tilde t_1}^2}{Q^2}
$$

$$
- 2\,(m_{\tilde g}^2 - m_t^2) \ln \frac{m_{\tilde g}^2}{m_t^2} \ln \frac{m_{\tilde t_1}^2}{Q^2} - (m_{\tilde t_1}^2 + m_{\tilde t_2}^2) \ln \frac{m_t^2\, m_{\tilde g}^2}{Q^4} \ln \frac{m_{\tilde t_1}^2}{Q^2} \Bigg]
$$

$$
- \Bigg[\left(\frac{8}{s_{2\theta}} - 12\, s_{2\theta} \right) \frac{m_t\, \Big(2\,\Delta + (m_{\tilde g}^2 + m_t^2 - m_{\tilde t_1}^2)\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2) \Big)}{m_{\tilde g}\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)^2} + \frac{4\,\Delta + 8\, m_{\tilde g}^2\, m_t^2}{m_{\tilde g}^2\,(m_{\tilde t_1}^2 - m_{\tilde t_2}^2)}
$$

$$
+ \frac{2\,(m_{\tilde g}^2 + m_t^2 - m_{\tilde t_1}^2)}{m_{\tilde g}^2} - \frac{4\, m_t^2\,(m_{\tilde g}^2 + m_t^2 - m_{\tilde t_1}^2 - 2\, m_{\tilde g}\, m_t\, s_{2\theta})}{\Delta} \Bigg] \, \Phi\,(m_t^2, m_{\tilde t_1}^2, m_{\tilde g}^2)\,, \tag{A3}
$$

where $\Delta = m_{\tilde g}^4 + m_t^4 + m_{\tilde t_1}^4 - 2\,(m_{\tilde g}^2\, m_t^2 + m_{\tilde g}^2\, m_{\tilde t_1}^2 + m_t^2\, m_{\tilde t_1}^2)$, and the function Φ is defined as in [20]:

$$\Phi\,(x,y,z) = \frac{1}{\lambda}\left(2\,\ln x_+\,\ln x_- - \ln u\,\ln v - 2\left(\mathrm{Li}_2\,x_+ + \mathrm{Li}_2\,x_-\right) + \frac{\pi^2}{3}\right)\,, \tag{A4}$$

where the auxiliary (complex) variables are:

$$u = \frac{x}{z}\,,\qquad v = \frac{y}{z}\,,\qquad \lambda = \sqrt{(1-u-v)^2 - 4\,u\,v}\,,\qquad x_\pm = \frac{1}{2}\left(1 \pm (u-v) - \lambda\right). \tag{A5}$$

The definition (A4) is valid for the case $x/z < 1$ and $y/z < 1$. The other branches of Φ can be obtained using the symmetry properties:

$$\Phi\,(x,y,z) = \Phi\,(y,x,z)\,,\qquad x\,\Phi\,(x,y,z) = z\,\Phi\,(z,y,x)\,. \tag{A6}$$

Appendix B: Shifts of the parameters to the on–shell scheme

In the OS renormalization scheme, the masses of all particles are defined as the poles of the corresponding propagators. As an example, for a scalar particle with squared mass m^2 the relation between the $\overline{\rm DR}$ and OS definitions of the mass is:

$$\delta m^2 = (m^2)^{\overline{\rm DR}} - (m^2)^{\rm OS} = \mathrm{Re}\,\hat\Pi\,(m^2)\,, \tag{B1}$$

where $\hat\Pi\,(m^2)$ is the finite part of the self-energy of the particle, evaluated at an external momentum equal to the mass itself. In the following we list the shifts to the on–shell scheme for the top and stop masses [1]:

$$\begin{aligned}
\frac{\delta m_t}{m_t} &= \frac{g_s^2}{16\pi^2}\,C_F\,\Bigg\{3\,\ln\frac{m_t^2}{Q^2} + \delta + \frac{m_{\tilde g}^2}{m_t^2}\left(\ln\frac{m_{\tilde g}^2}{Q^2} - 1\right) - \frac{1}{2}\Bigg[\frac{m_{\tilde t_1}^2}{m_t^2}\left(\ln\frac{m_{\tilde t_1}^2}{Q^2} - 1\right)\\
&\quad - \frac{m_{\tilde g}^2 + m_t^2 - m_{\tilde t_1}^2 - 2\,s_{2\theta}\,m_{\tilde g}\,m_t}{m_t^2}\,\hat B_0(m_t^2, m_{\tilde g}^2, m_{\tilde t_1}^2) + (1, s_{2\theta}) \leftrightarrow (2, -s_{2\theta})\Bigg]\Bigg\}\,, && \text{(B2)}\\
\frac{\delta m_{\tilde t_1}^2}{m_{\tilde t_1}^2} &= \frac{g_s^2}{16\pi^2}\,C_F\,\Bigg\{3\,\ln\frac{m_{\tilde t_1}^2}{Q^2} - 7 - c_{2\theta}^2\left(\ln\frac{m_{\tilde t_1}^2}{Q^2} - 1\right) - s_{2\theta}^2\,\frac{m_{\tilde t_2}^2}{m_{\tilde t_1}^2}\left(\ln\frac{m_{\tilde t_2}^2}{Q^2} - 1\right)\\
&\quad + 2\Bigg[\frac{m_{\tilde g}^2}{m_{\tilde t_1}^2}\left(\ln\frac{m_{\tilde g}^2}{Q^2} - 1\right) + \frac{m_t^2}{m_{\tilde t_1}^2}\left(\ln\frac{m_t^2}{Q^2} - 1\right)\\
&\quad + \frac{m_{\tilde t_1}^2 - m_{\tilde g}^2 - m_t^2 + 2\,s_{2\theta}\,m_{\tilde g}\,m_t}{m_{\tilde t_1}^2}\,\hat B_0(m_{\tilde t_1}^2, m_t^2, m_{\tilde g}^2)\Bigg]\Bigg\}\,, && \text{(B3)}\\
\frac{\delta m_{\tilde t_2}^2}{m_{\tilde t_2}^2} &= \frac{\delta m_{\tilde t_1}^2}{m_{\tilde t_1}^2}\quad \left[(1, s_{2\theta}) \leftrightarrow (2, -s_{2\theta})\right]\,, && \text{(B4)}
\end{aligned}$$

where the notation $(1, s_{2\theta}) \leftrightarrow (2, -s_{2\theta})$ in eq.(B2) means a term that is obtained from the previous ones inside the square bracket with the exchange $m_{\tilde t_1}^2 \leftrightarrow m_{\tilde t_2}^2$ and the replacement $s_{2\theta} \to -s_{2\theta}$.

[1] Similar results have been presented in [21, 11]. Formulae for the $\overline{\rm DR}$–OS shifts, specialized to the case $m_L^2 = m_R^2$, can also be found in [16]. However, we disagree with [16] on the contributions to the stop self–energies coming from the diagrams that involve the $\mathcal{O}(\alpha_s)$ four–squark vertex.

The notation of Eq. (B4) implies that $\delta m^2_{\tilde{t}_2}/m^2_{\tilde{t}_2}$ can be obtained from the right hand side of eq. (B3) with the above substitutions. The quantity δ that appears in eq. (B2) is a constant that depends on the regularization. In dimensional regularization $\delta = -4$, while in dimensional reduction $\delta = -5$. In eqs. (B2)–(B4), $\hat{B}_0$ denotes the finite part of the Passarino-Veltman function, i.e. :

$$\hat{B}_0\,(p^2, m_1^2, m_2^2) \;=\; -\int_0^1 dx \ln \frac{(1-x)\,m_1^2 + x\,m_2^2 - x\,(1-x)\,p^2 - i\epsilon}{Q^2}\,. \tag{B5}$$

An explicit expression for $\hat{B}_0$ can be found e.g. in [22].

Due to the relative freedom in the choice of the renormalization conditions for the stop sector, several definitions are possible for the shift in the mixing angle $\theta_{\tilde{t}}$ (for a discussion on this point, see [23] and references therein). We choose the following "symmetrical" definition:

$$\delta\theta_{\tilde{t}} = \frac{1}{2}\,\frac{\hat{\Pi}_{12}(m^2_{\tilde{t}_1}) + \hat{\Pi}_{12}(m^2_{\tilde{t}_2})}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\,, \tag{B6}$$

where $\hat{\Pi}_{12}(p^2)$ is the off–diagonal self–energy of the stops:

$$\begin{aligned}\hat{\Pi}_{12}(p^2) \;=\; \frac{g_s^2}{16\pi^2}\,C_F\,\Big\{&4\,m_t\,m_{\tilde{g}}\,c_{2\theta}\,\hat{B}_0\,(p^2, m_t, m_{\tilde{g}})\\ &+c_{2\theta}\,s_{2\theta}\left[m^2_{\tilde{t}_1}\left(\ln\frac{m^2_{\tilde{t}_1}}{Q^2} - 1\right) - m^2_{\tilde{t}_2}\left(\ln\frac{m^2_{\tilde{t}_2}}{Q^2} - 1\right)\right]\Big\}\,.\end{aligned} \tag{B7}$$

Finally, taking into account that μ and $\tan\beta$ do not get any $\mathcal{O}\,(\alpha_s)$ correction, the shift for the soft term A_t can be easily derived from the (field–independent) definition of $s_{2\theta}$, eq. (19):

$$\delta A_t = \left(\frac{\delta m^2_{\tilde{t}_1} - \delta m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} + \frac{\delta s_{2\theta}}{s_{2\theta}} - \frac{\delta m_t}{m_t}\right)\,(A_t + \mu\cot\beta)\,, \tag{B8}$$

where $\delta s_{2\theta}/s_{2\theta} \,=\, 2\,\cot 2\theta_{\tilde{t}}\,\delta\theta_{\tilde{t}}$. Eq. (B8) can be treated as a definition of A_t in our on–shell scheme.

Appendix C: Two-loop corrections to the CP-odd Higgs mass

We present here the two–loop $\mathcal{O}(\alpha_t\alpha_s)$ corrections to the mass of the CP–odd Higgs boson, A. As apparent from eq. (6), m_A^2 depends on the value of the soft supersymmetry–breaking parameter m_3^2, thus we included it among the MSSM input parameters. However, a calculation of the loop corrections to the relation between m_A^2, m_3^2 and $\tan\beta$ may be useful for discussing models that predict the values of the soft supersymmetry–breaking masses, and in particular m_3^2.

Starting from eqs. (6) and (9), and following the same line of reasoning as in Section 3, we find that:

$$m_A^2 = \frac{1}{\sin\beta\,\cos\beta}\left(-m_3^2 + \frac{h_t^2\,\mu\,A_t}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\,F_A\right)\,, \tag{C1}$$

where m_A is the $\mathcal{O}(\alpha_t\,\alpha_s^n)$ loop–corrected A mass evaluated in the effective potential approach, i.e. neglecting the corrections that depend on the external momenta. The function F_A can be

decomposed as $F_A = \widetilde{F}_A + \Delta\widetilde{F}_A$, where $\Delta\widetilde{F}_A$ contains terms coming from the renormalization of the parameters that multiply F_A in eq. (C1), and $\widetilde{F}_A$ is defined as:

$$\widetilde{F}_A = \frac{\partial V}{\partial m^2_{\tilde{t}_2}} - \frac{\partial V}{\partial m^2_{\tilde{t}_1}} + \frac{4\, c^2_{2\theta}}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \frac{\partial V}{\partial c^2_{2\bar{\theta}}} - \frac{2\,\mu\,\cot\beta\,(s_{2\theta})^{-2}}{A_t\,(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})} \frac{\partial V}{\partial\, c_{\varphi\tilde{\varphi}}}\,. \tag{C2}$$

The definitions of the field–dependent parameters $m^2_{\tilde{t}_1}$, $m^2_{\tilde{t}_2}$, $c^2_{2\bar{\theta}}$ and $c_{\varphi\tilde{\varphi}}$ are given in Section 3. The one–loop $\mathcal{O}(\alpha_t)$ contribution to F_A is known [6]:

$$F_A^{1\,\ell} = \frac{N_c}{16\,\pi^2}\left[m^2_{\tilde{t}_1}\left(1 - \ln\frac{m^2_{\tilde{t}_1}}{Q^2}\right) - m^2_{\tilde{t}_2}\left(1 - \ln\frac{m^2_{\tilde{t}_2}}{Q^2}\right)\right]\,. \tag{C3}$$

Assuming a $\overline{\rm DR}$ renormalization for the parameters h_t, A_t, $m^2_{\tilde{t}_1}$ and $m^2_{\tilde{t}_2}$ that enter in eqs. (C1) and (C3), the two–loop $\mathcal{O}(\alpha_t\,\alpha_s)$ contribution to F_A in the $\overline{\rm DR}$ renormalization scheme is given, in units of $g_s^2\,C_F\,N_c/(16\,\pi^2)^2$, by:

$$\begin{aligned}
\hat{F}_A^{2\,\ell} \;=\;& \frac{16\,\pi^2}{N_c}\,F_A^{1\,\ell}\left[8 - s^2_{2\theta}\left(2 - \frac{m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2}}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\,\ln\frac{m^2_{\tilde{t}_1}}{m^2_{\tilde{t}_2}}\right)\right]\\
&+2\left(m^2_{\tilde{t}_1}\,\ln^2\frac{m^2_{\tilde{t}_1}}{Q^2} - m^2_{\tilde{t}_2}\,\ln^2\frac{m^2_{\tilde{t}_2}}{Q^2}\right) + \frac{2}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\left(m^2_{\tilde{t}_1}\,\ln\frac{m^2_{\tilde{t}_1}}{Q^2} - m^2_{\tilde{t}_2}\,\ln\frac{m^2_{\tilde{t}_2}}{Q^2}\right)^2\\
&+ f_A(m_t\,, m_{\tilde{g}}\,, m^2_{\tilde{t}_1}\,, m^2_{\tilde{t}_2}\,, s_{2\theta}\,, Q) \;-\; f_A(m_t\,, m_{\tilde{g}}\,, m^2_{\tilde{t}_2}\,, m^2_{\tilde{t}_1}\,, -s_{2\theta}\,, Q)\,.
\end{aligned} \tag{C4}$$

The function f_A contains contributions coming from the top-stop-gluino diagrams (fig. 1d), and its explicit expression in units of $g_s^2\,C_F\,N_c/(16\,\pi^2)^2$ is:

$$\begin{aligned}
f_A(m_t\,, m_{\tilde{g}}\,, m^2_{\tilde{t}_1}\,, m^2_{\tilde{t}_2}\,, s_{2\theta}\,, Q) \;=\;& \frac{16\,m^2_{\tilde{t}_1}\,m_{\tilde{g}}\,m_t\,s_{2\theta}}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} - \frac{\pi^2\,m_{\tilde{g}}\,(m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})}{6\,A_t} - 4\left(m^2_{\tilde{g}} + m^2_t\right)\ln\frac{m^2_{\tilde{t}_1}}{Q^2}\\
&-\frac{2\,m_{\tilde{g}}}{A_t}\left[m^2_{\tilde{t}_1}\left(6 - 5\,\ln\frac{m^2_{\tilde{t}_1}}{Q^2}\right) + m^2_{\tilde{t}_2}\left(1 - \ln\frac{m^2_{\tilde{t}_2}}{Q^2}\right)\right] - \frac{4\,m_{\tilde{g}}\,m_t\,s_{2\theta}\,(3\,m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\,\ln\frac{m^2_{\tilde{t}_1}}{Q^2}\\
&-\frac{m_{\tilde{g}}}{A_t}\left(m^2_{\tilde{t}_1}\,\ln^2\frac{m^2_{\tilde{t}_1}}{Q^2} + m^2_{\tilde{t}_2}\,\ln^2\frac{m^2_{\tilde{t}_2}}{Q^2}\right) + 2\,(m^2_{\tilde{g}} + m^2_t - m^2_{\tilde{t}_1})\,\ln\frac{m^2_{\tilde{g}}\,m^2_t}{Q^4}\,\ln\frac{m^2_{\tilde{t}_1}}{Q^2}\\
&+2\,m^2_{\tilde{t}_1}\left(1 + \frac{m_{\tilde{g}}}{A_t}\right)\ln\frac{m^2_{\tilde{g}}}{Q^2}\,\ln\frac{m^2_t}{Q^2} - \frac{2\,m_{\tilde{g}}}{A_t}\left[(m^2_{\tilde{g}} - m^2_t)\ln\frac{m^2_{\tilde{g}}}{m^2_t} + m^2_{\tilde{t}_1}\,\ln\frac{m^2_{\tilde{g}}\,m^2_t}{Q^4}\right]\ln\frac{m^2_{\tilde{t}_1}}{Q^2}\\
&+\frac{2\,m_{\tilde{g}}\,m_t\,s_{2\theta}}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}}\left[2\,(m^2_{\tilde{g}} - m^2_t)\ln\frac{m^2_{\tilde{g}}}{m^2_t} + (m^2_{\tilde{t}_1} + m^2_{\tilde{t}_2})\,\ln\frac{m^2_{\tilde{g}}\,m^2_t}{Q^4}\right]\ln\frac{m^2_{\tilde{t}_1}}{Q^2}\\
&-\left[4\,m^2_t + \frac{2\,\Delta}{m^2_{\tilde{g}}}\left(1 + \frac{m_{\tilde{g}}}{A_t}\right) - \frac{2\,m_t\,s_{2\theta}\left(2\,\Delta + (m^2_{\tilde{g}} + m^2_t - m^2_{\tilde{t}_1})\,(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})\right)}{m_{\tilde{g}}\,(m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2})}\right]\Phi(m^2_t, m^2_{\tilde{t}_1}, m^2_{\tilde{g}})\,,
\end{aligned} \tag{C5}$$

where Δ and $\Phi(x, y, z)$ are defined at the end of appendix A.

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